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Gravity Illustrated

Spacetime Edition

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Abstract

This thesis deals with essentially four different topics within general relativity: pedagogical techniques for illustrating curved spacetime, inertial forces, gyroscope precession and optical geometry. Concerning the pedagogical techniques, I investigate two distinctly different methods, the *dual* and the *absolute* method.

In the dual scheme, I start from the geodesic equation in a 1+1 static, diagonal, Lorentzian spacetime, such as the Schwarzschild radial line element. I then find another metric, with Euclidean signature, which produces the same geodesics $x(t)$. This geodesically equivalent *dual* metric can be embedded in ordinary Euclidean space. Freely falling particles correspond to straight lines on the embedded surface.

In the absolute scheme, I start from an arbitrary Lorentzian spacetime with a given field of timelike four-velocities u^{μ}. I then perform a coordinate transformation to the local Minkowski system comoving with the given four-velocity at every point. In the local system the sign of the spatial part of the metric is flipped to create a new metric of Euclidean signature. For the particular case of two dimensions we may embed this *absolute* geometry as a curved surface. The method is well suited for visualizing gravitational time dilation, cosmological expansion and black holes.

Concerning inertial forces, gyroscope precession and optical geometry, the general framework is based on the introduction of a congruence of reference worldlines in an arbitrary spacetime. This allows us to describe the local motion and acceleration of particles in terms of the speed relative to the congruence, the time derivative of the speed and the spatial curvature (project down along the reference congruence) of the corresponding worldline.

I present two papers concerning inertial forces in this framework, one formal and one intuitive. I also present two papers concerning gyroscope precession, again one formal and one intuitive. In particular I illustrate how one can explain gyroscope precession in an arbitrary stationary spacetime as a double Thomas precession effect.

Introducing a novel type of spatial curvature measure for the worldline of a test particle, we present a natural way of generalizing the theory of optical geometry to include arbitrary spacetimes. The generalized optical geometry allows us to do optical geometry across the horizon of a black hole.

Keywords: curved spacetime, embeddings, pedagogical techniques, inertial forces, gyroscope precession, optical geometry